Performance analysis of linear multiuser detectors on a Nakagami-m fading channel

Tarciana Lopes, Renato Baldini, Celso de Almeida and Rodrigo Ramos *

Abstract

The aim of this work is to analyze the performance of linear multiuser detectors in a synchronous CDMA system, on a Nakagami-m fading channel. Simplified expressions to evaluate the performance of the decorrelating detector and the Minimum Mean Square Error (MMSE) detector, using random spreading sequence on a Nakagami-m channel, are obtained. For both detectors, bit error rate (BER) analytical expressions are obtained and they are validated by simulations. These expressions take into account diversity L and an integer range of values of m. This work also presents a comparison between a n-path Rayleigh channel and a n-order Nakagami channel.
Keywords - Multiuser detection, decorrelating detector, multiple access interference, Nakagami-m fading.

1 Introduction

In CDMA systems, all transmitters share the same channel at the same time. Thus, at the reception, all signals are overlapped in time and frequency which cause mutual interferences. Those interferences, known as multiple access interference (MAI) [1], are the result of random time offsets among the signals, that makes unfeasible the code waveforms to be completely orthogonal.

As the number of interferers or their amplitudes increase, the MAI experienced by a given user also increases. This means that, if the systems are loaded and/or there are significant differences among the received signal powers (near-far effect), the problems with the interfering signals are critical. Those problems, however, can be mitigated by employing techniques like power control, efficient spreading codes, smart antennas and multiuser detection (MUD).

In view of the benefits of the multiuser detection, that exploits the inherent structure of the MAI and not just considers it as additive noise, this study investigates CDMA systems, in terms of mean bit error probability, using a linear multiuser detector in the reception.

Two kinds of detectors are investigated, decorrelating and MMSE detectors. The channel model is the Nakagami-m fading channel. Analytical expressions of the bit error probability, for both detectors, are obtained and simulations are performed to assert the results.

This letter is organized as follows. In Section II we present the system model, in Section III the multiuser detectors are described. Their performances are analyzed in section IV. Numerical results are presented in Section V and conclusions are in Section VI.

2 System model

It is considered a synchronous CDMA system with K simultaneous users. An unicellular environment with perfect power control, random spreading sequences for each user and BPSK (binary phase-shift keying) modulation is assumed. The signals are transmitted on a channel with multipath fading. This fading is modelled by a Nakagami-m distribution.

The received signal is the sum of antipodal modulated synchronous signature waveforms embedded in additive white gaussian noise, that can be expressed by:

$$r(t) = \sum_{k=1}^{K} \alpha_k A_k b_k s_k(t) + \sigma n(t), \qquad t \ \epsilon \ [0, T_b) \quad (1)$$

where: T_b is the symbol period (bit); $b_k \ \epsilon \ \{+1, -1\}$ is the bit transmitted by the k^{th} user; α_k is the fading coefficient for the user k, assumed to be a random variable with Nakagami-m distribution [2]. A_k is the amplitude of the k^{th} users signal. The energy of the k^{th} signal is expressed by $E_b = A_k^2 T_b$; $n(t)$ is the white gaussian noise with unit power spectral density. And $s_k(t)$ is the signature waveform assigned to the k^{th} user. The rate of the code waveform, ($f_c = 1/T_c$), is greater than the bit rate ($f_b = 1/T_b$). Thus, multiplying the BPSK signal at the transmitter by $s_k(t)$ has the effect of spreading it out in frequency by a factor of $G_p = T_b/T_c$, defined as the processing gain. The signature waveforms are assumed to be zero outside the interval $[0, T_b]$, so there is no intersymbol interference (ISI).

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3 Multiuser Detection

There are several realizations of an optimum detector for DS-CDMA systems, but all usually require a high computacional complexity. For instance, the complexity of the maximun likehood sequence (MLS) detector [1] increases exponentially with the number of users. A class of suboptimum detectors with low complexity is the class of linear detectors. The linear multiuser detectors apply a linear mapping to the soft output of a convencional detector, or matched filter, to reduce the MAI for each user. In this section we briefly review two linear detectors, the decorrelator and the minimum mean-squared error (MMSE) detector.

3.1 Decorrelating Detector

The decorrelator, the knowledge of the received amplitudes is not required and the same degree of robustness of the optimum detector against imbalances in the received amplitudes can be achieved. The decorrelating detector achieves the maximum near-far resistance with a computacional complexity per modulated bit similar to that of the single-user matched filter. Moreover, the decorrelator is resistant to MAI.

Fig. 1 illustrates the decorrelating detector structure.

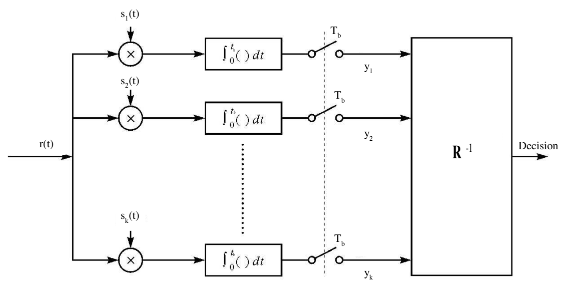

Figure 1: The decorrelating detector structure.

The received signal, as defined in Eq. (1), can be expressed in matrix notation as:

$$\mathbf{r} = \mathbf{GAb} + \mathbf{n}, \quad (2)$$

where $\mathbf{G}$ is a $K \times G_p$ matrix, whose entries correspond to the spreading sequences, $\mathbf{A}$ is a K-diagonal matrix containing the corresponding received amplitudes and $\mathbf{b}$ and $\mathbf{n}$ are K-vectors that hold the transmitted bits and the noise, respectively.

These received signal is, then, introduced in a bank of matched filters, where each code waveform is regenerated and correlated with the received signal in a separate detector branch. The matched filter outputs, expressed in matrix form, are

$$\mathbf{y} = \mathbf{G}^T\mathbf{r}, \quad (3)$$

where $\mathbf{y} = [y_1, \ldots, y_k]$ and $\{.\}^T$ means the transpose operation. Replacing (2) in (3):

$$\mathbf{y} \;=\; \mathbf{RAb} + \mathbf{G}^T\mathbf{n}, \quad (4)$$

where $\mathbf{R} = \mathbf{G}^T\mathbf{G}$ is the cross-correlation matrix, whose entries are the pairwise cross-correlations of the spreading codes , expressed by:

$$\rho_{kj} = \int_0^{T_b} s_k(t)s_j(t)dt. \quad (5)$$

The term $\mathbf{G}^T\mathbf{n}$ is a zero-mean gaussian random vector with covariance matrix defined by $E[\mathbf{G}^T\mathbf{n}\mathbf{n}^T\mathbf{G}] = \sigma^2\mathbf{R}$.

If the code waveforms are known, at the receiver, the matrix $\mathbf{R}$ can be assembled and its inverse can be evaluated. Then, multiplying $\mathbf{R^{-1}}$ by $\mathbf{y}$, results:

$$\mathbf{z} = \mathbf{R}^{-1}\mathbf{y} = \mathbf{Ab} + \mathbf{R}^{-1}\mathbf{n}. \quad (6)$$

Notice that the user signals are decorrelated from each other, i.e., the k^{th} component of $\mathbf{z}$ is free from interference caused by any other user. Thus, the near-far effect is also eliminated and the power control is no long necessary. The source of interference is, therefore, restricted to the background noise.

The decorrelator does not require the knowledge of the received amplitudes, thus it is not sensitive to estimation errors. It has computational complexity significantly lower than the MLS detector. The per-bit complexity of the decorrelator is linear with the number of users, excluding the cost of recomputation of the inverse mapping. Besides, it corresponds to the MLS detector [1] when the energies of all users are not known at the receiver. The disadvantage of this detector is the noise enhancement, because the norm of $\mathbf{R}$ is not necessarily lesser than or equal to one. Despite of this drawback, the decorrelator provides generally significant improvements over the convencional detector.

3.2 MMSE Detector

The MMSE detector evaluates a modified inverse of the correlation matrix, $\mathbf{R}$, taking into account the background noise. This detector implements a linear mapping which minimizes the mean-squared error between k^{th} users bit and the output of the k^{th} linear transformation $\mathbf{m}_k^T\mathbf{y}$, i.e., it is chosen the K-vector $\mathbf{m}$ that minimizes [1]:

$$E\left[\left(b_k - \mathbf{m}_k^T\mathbf{y}\right)^2\right]. \quad (7)$$

The linear transformation which achieves MMSE is:

$$\mathbf{M} = [\mathbf{R} + (\sigma\mathbf{A}^{-1})^2]^{-1}. \quad (8)$$

The soft estimate for the k^{th} user is the k^{th} component of $\mathbf{My}$, i.e.:

$$\hat{b}_k = \mathrm{sgn}\left((\mathbf{My})_k\right). \quad (9)$$

As the background noise goes to zero, and thus, the signal-to-noise ratio goes to infinity, the MMSE detector has the same performance of the decorrelator:

$$\lim_{\sigma\to 0}[\mathbf{R} + (\sigma\mathbf{A}^{-1})^2]^{-1} = \mathbf{R}^{-1}. \qquad (10)$$

On the other hand, if $\sigma \to \infty$, $[\mathbf{R} + (\sigma\mathbf{A}^{-1})^2]^{-1}$ becomes a diagonal matrix, and the MMSE detector approaches the convencional matched filter.

The MMSE detector structure is quite similar to Fig. 1, but the linear transformation evaluated is M, instead of R^{-1}.

Unlike the decorrelator, the MMSE detector requires estimation of the received amplitudes and its performance depends on the power of the interfering signals. Thus, it presents smaller near-far resistance if compared to the decorrelating detector.

Whichever multiuser technique is used, a substantial computation is required. Therefore, the use of the same short code, for each bit, is assumed [5]. Then, the partial correlation between all signals are the same for each bit, what minimizes the need for recomputation of the matrix $\mathbf{R}$ and, consequently, its inverse, from one bit interval to the next.

4 Performance of MUD on Nakagami-m channels

4.1 Decorrelating Detector

The bit error probability for the decorrelator on a Nakagami-m fading channel, using random spreading sequences, is evaluated in this sub-section.

The bit error probability on a flat-fading channel, for the k^{th} user is given by [1]:

$$P_b = Q\left(\sqrt{\frac{2\gamma_b}{R^+_{kk}}}\right), \qquad (11)$$

where R^+_{kk} represents $(R^{-1})_{kk}$, $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}} dt$ and $\gamma_b = \alpha^2 \frac{E_b}{N_0}$ is the signal-to-noise ratio (SNR). Once α has a Nakagami-m distribution, γ_b has a PDF given by:

$$p(\gamma_b) = \frac{m^m}{\Gamma(m)\overline{\gamma_b}^m} \gamma_b^{m-1} e^{(-m\frac{\gamma_b}{\overline{\gamma_b}})}, \qquad (12)$$

where $\overline{\gamma_b} = E[\alpha^2]\frac{E_b}{N_0}$, $\Gamma(.)$ is the Gamma function expressed by:

$$\Gamma(m) = \int_0^\infty x^{m-1} e^{-x} dx, \qquad (13)$$

and m is the fading figure defined by:

$$m = \frac{\Omega^2}{E[(r^2-\Omega)^2]}, \qquad m \geq \frac{1}{2}, \qquad (14)$$

where $\Omega = E[r^2]$.

Unconditioning Eq. (11) in relation to the Eq. (12) results:

$$P_{b_k} = \frac{2m^m}{\Gamma(m)\overline{\gamma_b}^m} \int_0^\infty Q\left(\frac{x}{\delta}\right) x^{2m-1} e^{(\frac{-mx^2}{\overline{\gamma_b}})} dx, \qquad (15)$$

where $x = \sqrt{\gamma_b}$ and $\delta = \sqrt{\frac{R^+_{kk}}{2}}$.

Solving the integration results:

$$P_{b_k} = \frac{(\frac{\overline{\gamma_b}}{mR^+_{kk}})^{-m}\Gamma(m+\frac{1}{2})\,{}_2F_1([m, m+\frac{1}{2}], m+1, \frac{-mR^+_{kk}}{\overline{\gamma_b}})}{2\Gamma(m)\, m\, \sqrt{\pi}} \qquad (16)$$

where ${}_2F_1([\cdot,\cdot]\cdot,\cdot)$ represents a Hypergeometric function with the following series representation:

$${}_2F_1([a,b], c, z) = \sum_{j=0}^{\infty} \frac{(a)_j (b)_j}{(c)_j} \frac{z^k}{k!}, \qquad (17)$$

where $(x)_j = x(x+1)(x+2)\cdots(x+j)$.

Analyzing the properties of the $Q(\cdot)$ function, an expression for the bit error probability, for a integer range of values of m, is obtained:

$$P_{b_k} = \left[\frac{1}{2}(1-\mu)\right]^m \sum_{j=0}^{m-1} \binom{m-1+j}{j} \left[\frac{1}{2}(1+\mu)\right]^j, \qquad (18)$$

where μ is given by:

$$\mu = \sqrt{\frac{1}{1+\frac{mR^+_{kk}}{\overline{\gamma_b}}}}. \qquad (19)$$

Observe that Eq.(18) is equivalent to the the bit error probability expression for a system with diversity on a Rayleigh fading channel. For a Nakagami-m distribution with $m = 2$, the performance of the decorrelating detector is identical to the performance obtained on a Rayleigh fading channel with diversity $L = 2$. Generalizing, a Nakagami-m fading channel with no diversity and integer value parameter m corresponds to a Rayleigh fading channel with diversity $L = m$. Moreover, a d-order diversity system transmitting on a Nakagami-m fading channel with independent fading is equivalent to a $L = dm$ diversity on a Rayleigh fading channel.

Simulations were carried out to validate the equivalence between the decorrelator performance on a Rayleigh channel with diversity L and on a Nakagami-L fading channel. The discrete time model is represented by:

$$\mathbf{y}_l = \mathbf{R}(l)\mathbf{A}(l)\mathbf{b} + \mathbf{n}_l, \qquad (20)$$

where $\mathbf{A}(l) = diag\{A_{l1}, \ldots, A_{lK}\}$ and $\mathbf{n}_l$ is a gaussian vector with covariance matrix equal to $2\sigma^2\mathbf{R}(l)$.

Fig. 2 illustrates a multipath CDMA system using a decorrelating detector [6]. In the diagram, $\text{MF}_{k,l}$ represents the filter matched to the k^{th}-user on the l^{th}-path. The outputs of the correlators are combined in a RAKE receiver, according to a maximal ratio combining scheme (MRC). After that, the correlation matrix is inverted. Therefore, it is avoided to invert the $\mathbf{R}(l)$ for each one of the L paths.

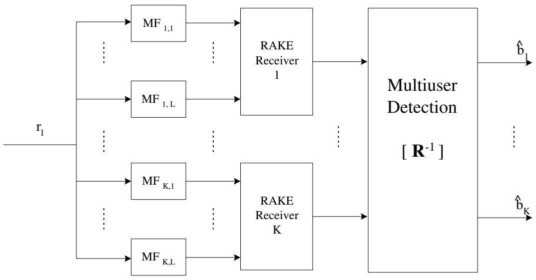

Figure 2: Diagram of a L-path CDMA system with a decorrelating detector.

The performance of the decorrelator on a Nakagami-L channel and on a Rayleigh channel are presented in section V.

4.2 MMSE Detector

Since the MMSE linear detector converges to the decorrelator for $\sigma \rightarrow 0$, its near-far resistance and asymptotic multiuser efficiency are identical to those of the decorrelator. The asymptotic multiuser efficiency η_k is defined as the ratio between the effective and actual energies [1]. For the synchronous CDMA case, it converges to $\eta_k = \frac{1}{R^+_{kk}}$.

The exact bit error probability expression, on a synchronous system for a given signal energy and cross-correlation, is given by Eq.(21) [1], where $\mathbf{M}$ is the MMSE linear mapping and $(\mathbf{MR})_{kk}$ represents the $(k,k)^{th}$ element of $\mathbf{MR}$. Moreover, the term $\left(\frac{A_k(\mathbf{MR})_{1k}}{(\mathbf{MRA})_{11}}\right)$ represents the contribution of the k^{th} interfering user to the decision variable. The user 1 is considered as the target. Notice that Eq. (21) is difficult to be solved, because the number of terms raises exponentially with K.

Thus, an approximation for the bit error probability is used, and random spreading sequences is considered. In a synchronous CDMA system with a MMSE detector, this approximation is given by [10]:

$$P_b \simeq Q\left(\sqrt{2\gamma_b - \frac{1}{4}\mathcal{F}\left(2\gamma_b, \frac{K-1}{G_p}\right)}\right), \quad K < G_p, \tag{22}$$

where γ_b is the system SNR and

$$\mathcal{F}(x,y) \triangleq \left(\sqrt{x(1+\sqrt{y})^2+1} - \sqrt{x(1-\sqrt{y})^2+1}\right)^2. \tag{23}$$

The bit error probability with perfect power control and using random spreading sequences, conditioned on the fading, is given by [10]:

$$P_b = Q\left(\sqrt{\sum_{j=1}^{m} \alpha_j^2 \left[2\frac{E_b}{N_0} - \frac{1}{4}\mathcal{F}\left(2\frac{E_b}{N_0}, \frac{K-1}{G_p}\right)\right]}\right). \tag{24}$$

The mean bit error probability is then given by averaging Eq. (24) over the Nakagami-m fading [3], i.e;

$$P_{b_k} = \int_0^\infty P_b \frac{m^m}{\Gamma(m)\overline{\gamma_b}^m} \gamma_b^{m-1} e^{(-m\frac{\gamma_b}{\overline{\gamma_b}})} \, d\gamma_b. \tag{25}$$

Evaluating the integral yields:

$$P_{b_k} = \frac{(\frac{\overline{\gamma_b}}{m})^{-m}\Gamma(m+\frac{1}{2})\ {}_2F_1([m, m+\frac{1}{2}], m+1, \frac{-m}{\overline{\gamma_b}})}{2\Gamma(m)\ m\ \sqrt{\pi}} \tag{26}$$

where $\overline{\gamma_b}$ is given by:

$$\overline{\gamma_b} = E[\alpha^2]\left[\frac{E_b}{N_0} - \frac{1}{8}\mathcal{F}\left(2\frac{E_b}{N_0}, \frac{K-1}{G_p}\right)\right]. \tag{27}$$

Similarly for the decorrelator, for integer values of m, the Eq.(25) is equivalent to that obtained for a Rayleigh fading channel with diversity of ordem m. Thus:

$$P_{b_k} = \left[\frac{1}{2}(1-\mu)\right]^m \sum_{j=0}^{m-1} \binom{m-1+j}{j} \left[\frac{1}{2}(1+\mu)\right]^j, \tag{28}$$

where μ is expressed by:

$$\mu = \sqrt{\frac{\overline{\gamma_b}}{\overline{\gamma_b}+m}}. \tag{29}$$

In order to verify the equivalence between the MMSE performance on a Nakagami-L and on a L-order diversity Rayleigh channels, a L-path environment was simulated. The diagram of a L-path system with a MMSE detector is quite similar to the one for the decorrelator, but instead of $\mathbf{R}^{-1}$, the linear transformation evaluated is $\mathbf{M}$. This is also computed once, and the matched filter outputs are combined by a maximal ratio scheme.

5 Numerical Results

The Monte Carlo method was performed. In the simulations, the bit error probability estimated is the ratio between the number of errors and the number of trials. For SNRs from 0 to 10 dB, each user transmits 10^5 bits, for SNRs from 10 to 15 dB each one transmits 10^6 bits. This guarantees a reliable approximation for the bit error probability.

The simulation model makes the channel gain a Nakagami-m distributed random variable. Channel estimation is assumed perfect. The system has $K = 10$ active users and each one experiences independent fading. The curves are obtained for a process gain $G_p = 32$.

$$P_{b_1} = 2^{K-1} \sum_{b_2,\cdots,b_k \in [-1,1]^{K-1}} Q\left(\frac{A_1(\mathbf{MR})_{11}}{\sigma\sqrt{(\mathbf{MRM})_{11}}}\left(1 + \sum_{k=2}^{K} \frac{A_k(\mathbf{MR})_{1k}}{(\mathbf{MRA})_{11}} b_k\right)\right). \quad (21)$$

5.1 Decorrelator

Fig. 3 presents the bit error probability of a synchronous CDMA system using the decorrelating detector in the reception, on a Nakagami-m channel. As the fading factor m increases, the fading becomes less severe, therefore, the bit error probability decreases. The curves are obtained by Eq. (18) and by simulation, and they show a good fit.

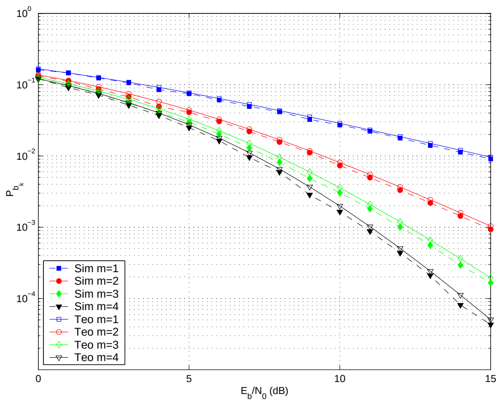

Figure 3: Decorrelator bit error probability on a Nakagami-m channel.

The comparisons between the performance of the decorrelator on a Nakagami-L channel and on a Rayleigh channel with diversity L are shown in Fig. 4. Observe the gain in performance due to the use of diversity.

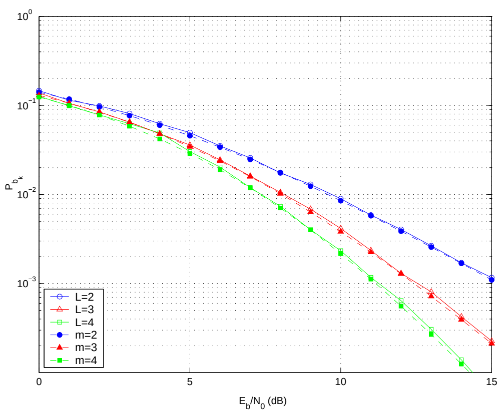

Figure 4: Performance of Decorrelator on Nakagami-L and on a L-path Rayleigh channels.

Fig. 5 illustrates the equivalence between systems with Rayleigh fading and diversity $L = md$ and systems with Nakagami-m fading and d-order diversity. It is worth to emphasize that the equivalence is only valid to signals combined by a maximal ratio scheme.

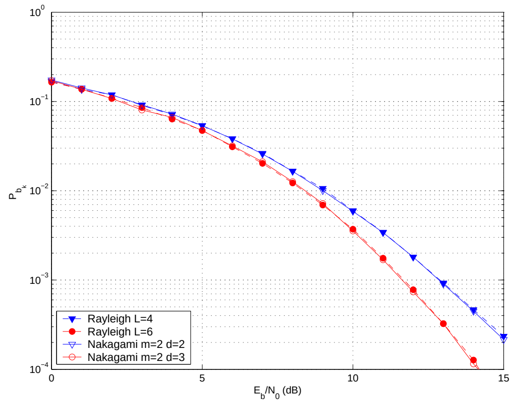

Figure 5: Performance of Decorrelator on Nakagami-m and on a Rayleigh multipath channels.

5.2 MMSE

Fig. 6 illustrates the performance of MMSE on a Nakagami-m fading channel. Simulations were carried out in order to validate the mean bit error probability expression. Eq. (28) and the simulation results are very similar.

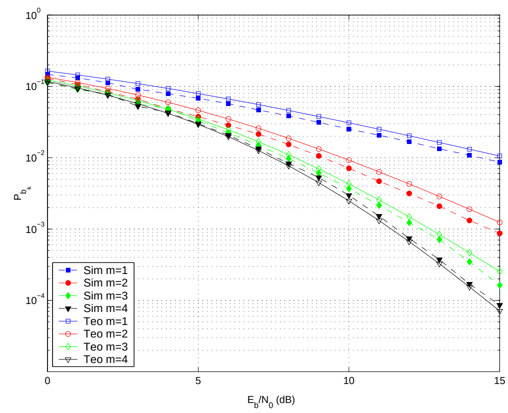

Figure 6: Performance of MMSE detector on a Nakagami-m channel.

The performance of a CDMA system using MMSE is illustrated in Fig. 7. Two configurations have been used: a Rayleigh fading channel with L-order diversity and a Nakagami-L fading channel. The bit error probability curves versus E_b/N_0 for both cases present good similarity.

The performance of systems with Rayleigh fading and diversity $L = md$ and systems with Nakagami-m fading and d-order diversity is shown in Fig. 8. Again, they are similar, as expected, and the equivalence is only valid for signals combined by a maximal ratio scheme.

The expressions obtained for both detectors are also effective in environments with medium and

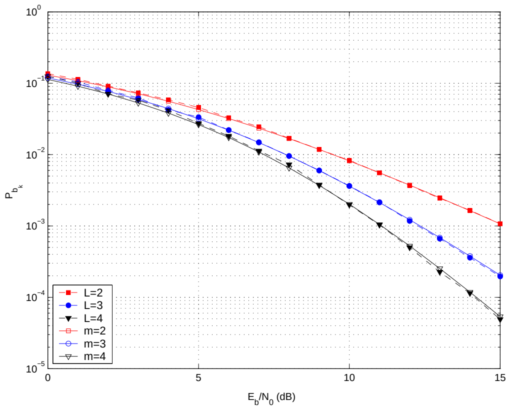

Figure 7: Performance of MMSE detector on Nakagami-L and on a L-path Rayleigh channels.

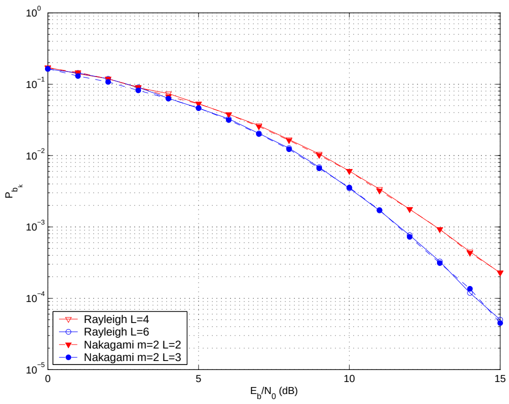

Figure 8: Performance of MMSE detector on Nakagami-m and on a Rayleigh multipath channels.

high load, as can be seen in Fig. 9. The process gain G_p is 32, fading figure m=3, SNR is 8 dB and different values of load, $(\frac{K-1}{G_p})$.

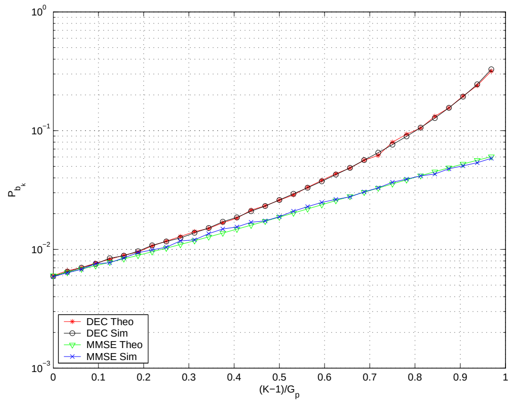

Figure 9: Performance of Decorrelator and MMSE detectors on Nakagami-m channel.

6 Conclusion

In this paper, it has been analyzed the performance of the multiuser decorrelating and MMSE detectors, on a Nakagami-m fading channel. The performance has been evaluated in terms of the mean bit error probability, using analytical expressions for both detectors. In order to validate the expressions obtained, simulations have been carried out. The simulations results have a high degree of agreement with the analytical expressions. These results confirm the equivalence between Rayleigh fading channel with diversity and Nakagami-m fading channel, for signals combined by a maximal ratio scheme. From this, it can clearly be seen how versatile is the Nakagami fading model.

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